Recent Advances in Modelling Masonry Shells: Validation and Application

Paulo B. Lourenço[1)]

Department of Civil Engineering, University of Minho, 4800 Guimarães, Portugal

ABSTRACT

A composite plasticity model for masonry shells is proposed. Modern algorithmic plasticity concepts - including implicit Euler backward return mapping schemes and consistent tangent operators for all regimes of the model - are utilised to combine anisotropic elastic behaviour with anisotropic plastic behaviour. The model is capable of reproducing independent (in the sense of completely diverse) elastic and inelastic behaviour along a prescribed set of material axes. An example of the performance of the model is presented, by means of a comparison between numerical results and experimental results for the case of one masonry panel with out-of-plane loading. Examples of the application of the model to solve real engineering problems are also shown.

INTRODUCTION

Masonry is a composite material made of units and joints and its modelling can individualise each unit and joint or represent just an averaged anisotropic continuum. The field of application of anisotropic continuum models is large structures, subjected to loads and boundary conditions such that the state of stress and strain across a macro-length can be assumed to be uniform. In reality, the material in non-homogeneous and a close material representation is only possible if the units and joints are modelled separately (Lourenço and Rots 1997). A macro-modelling strategy represents a compromise between efficiency and accuracy.

In this paper, the problem of modelling laminar masonry structures (plates and shells with one dimension substantially smaller than the other two dimensions) is addressed. A typical hypothesis in this type of elements is "zero normal stress". This hypothesis states that the normal stress component perpendicular to the plane of the structure equals zero, and simplifies

---

[1)] Assistant Professor

material modelling to a great extent. While, conceptually, it is relatively straightforward to include the three-dimensional behaviour in the model, one should keep in mind that the complexity of an anisotropic material model might preclude its numerical implementation and its use in practice, due to a large number of material parameters. The approach followed here is, basically, an engineering approach where a compromise is sought between accuracy and simplification.

PROPOSED MODEL

The difficulties in accurately modelling the behaviour of anisotropic materials are quite strong. Criteria such as those of Hill, Hoffman and Tsai-Wu have been proposed both from purely theoretical and experimental standpoints as failure criteria. But only a few numerical implementations and calculations have actually been carried out. Examples are given in (Owen and Figueiras 1983), (de Borst and Feenstra 1990), (Schellekens and de Borst 1990), (Swan and Cakmak 1994) and (Li *et al* 1994), which used crude representations of inelastic behaviour.

One serious problem associated with smooth criteria is the poor representation of materials with a large difference between uniaxial compressive strength and uniaxial tensile strength, which leads to unacceptable overestimation of strength in the tension-compression regime. To obtain a better representation, individual yield criteria must be considered, according to different failure mechanisms, one in tension and the other in compression. The former is associated with a localised fracture process, denoted by cracking of the material, and, the latter, is associated with a more distributed fracture process which is usually termed crushing of the material. The basis of the present paper is the plane stress composite criterion proposed in (Lourenço e*t al* 1997), see Figure 1.

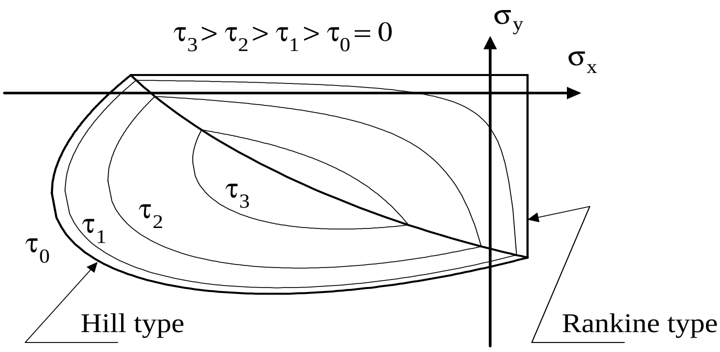


Fig. 1. Plane stress composite yield criterion with iso-shear stress lines (Lourenço et *al* 1997)

The curved shell element degenerated from a three-dimensional formulation is adopted in the analyses. Five degrees of freedom are defined for each element node: three translations and two rotations. For the sake of simplicity, the formulation of the model is presented based on the assumption that the principal axes of anisotropy coincided with the frame of reference (local or global) for stresses and strains in finite element computations. Since this is not necessarily the case, such non-alignment effects must be taken into account.

Tension - A Rankine Type Criterion

For modelling tensile behaviour, it will be assumed that cracks, at each integration point, always arise normal to the mid-surface of the element. This assumption means that each layer of the shell element is considered to be in plane stress and the additional stresses from the shell formulation (τ_{yz} and τ_{xz}) will be ignored. An adequate formulation of an anisotropic model with different tensile strengths along the *x*, *y* directions (Lourenço *et al* 1997), is

$$f_1 = \frac{(\sigma_x - \bar{\sigma}_{tx}(\kappa_t)) + (\sigma_y - \bar{\sigma}_{ty}(\kappa_t))}{2} + \sqrt{\left(\frac{(\sigma_x - \bar{\sigma}_{tx}(\kappa_t)) - (\sigma_y - \bar{\sigma}_{ty}(\kappa_t))}{2}\right)^2 + \alpha\ \tau_{xy}^2} = 0 \quad (1)$$

where $\bar{\sigma}_{tx}(\kappa_t)$ and $\bar{\sigma}_{ty}(\kappa_t)$ are, respectively, the tensile yield values along the material axes *x* and *y*, and the parameter α controls the shear stress contribution to failure. Exponential tensile softening is considered for both equivalent stress-equivalent strain diagrams.

Compression - A Hill Type Criterion

In case of crushing it is physically appealing and it results quite simple to include the contribution of the additional stresses from the shell formulation (τ_{yz} and τ_{xz}) in the failure criterion. The simplest yield criterion that features different compressive strengths along the two material axes is a rotated centred ellipsoid in the full stress space. The expression for such a quadric can be written as

$$f_2 = \frac{\bar{\sigma}_{cy}(\kappa_c)}{\bar{\sigma}_{cx}(\kappa_c)}\sigma_x^2 + \beta\sigma_x\sigma_y + \frac{\bar{\sigma}_{cx}(\kappa_c)}{\bar{\sigma}_{cy}(\kappa_c)}\sigma_y^2 + \gamma(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) - \bar{\sigma}_{cx}(\kappa_c)\,\bar{\sigma}_{cy}(\kappa_c) = 0 \qquad (2)$$

where $\bar{\sigma}_{cx}(\kappa_c)$ and $\bar{\sigma}_{cy}(\kappa_c)$ are, respectively, the compressive yield values along the material axes *x* and *y*. The β and γ values are additional material parameters that determine the shape of the yield criterion. The inelastic law adopted comprehends parabolic hardening followed by parabolic/exponential softening for both equivalent stress-equivalent strain diagrams.

Inelastic behaviour of the model

The behaviour of the model in uniaxial tension and compression along the material axes has been discussed in detail in (Lourenço *et al* 1997). Figure 2 shows the possibilities of the model under simple loading.

VALIDATION

The anisotropic continuum model for the analysis of masonry plates and shells has been validated for walls (Lourenço *et al* 1998) as well as plates and shells (Lourenço 1997). Here panel W2, with dimensions $5000 \times 2800 \times 150\ mm^3$ and tested by Gazzola *et al* (1985), has been selected to show the performance of the model The panel was loaded until failure with increasing out-of-plane uniform pressure *p*.

The response is typical of out-of-plane loaded panels, with cracking starting to occur at the bottom of the panel, Figure 3. Predominant cracking occurs in the shorter span direction, where higher bending moments and lower tensile strength can be found. This behaviour remains until peak load, with, practically, no diagonal cracking near the supports. At peak, a central crack parallel to the longer span crosses completely the panel. After peak load, cracks rapidly progress towards the corners of the panel and a yield-line type of collapse with marked softening lines is obtained, Figure 4. A secondary load path is formed with the slab arching through the long span. Thus, yield line analysis is not adequate for this masonry panel because softening lines are not formed at peak. A yield-line type of failure is retrieved only after peak, for substantially lower loads.

Figure 5 demonstrates the robustness of the techniques adopted to solve the non-linear problem as the full path of the response, up to complete loss of strength, could be traced.

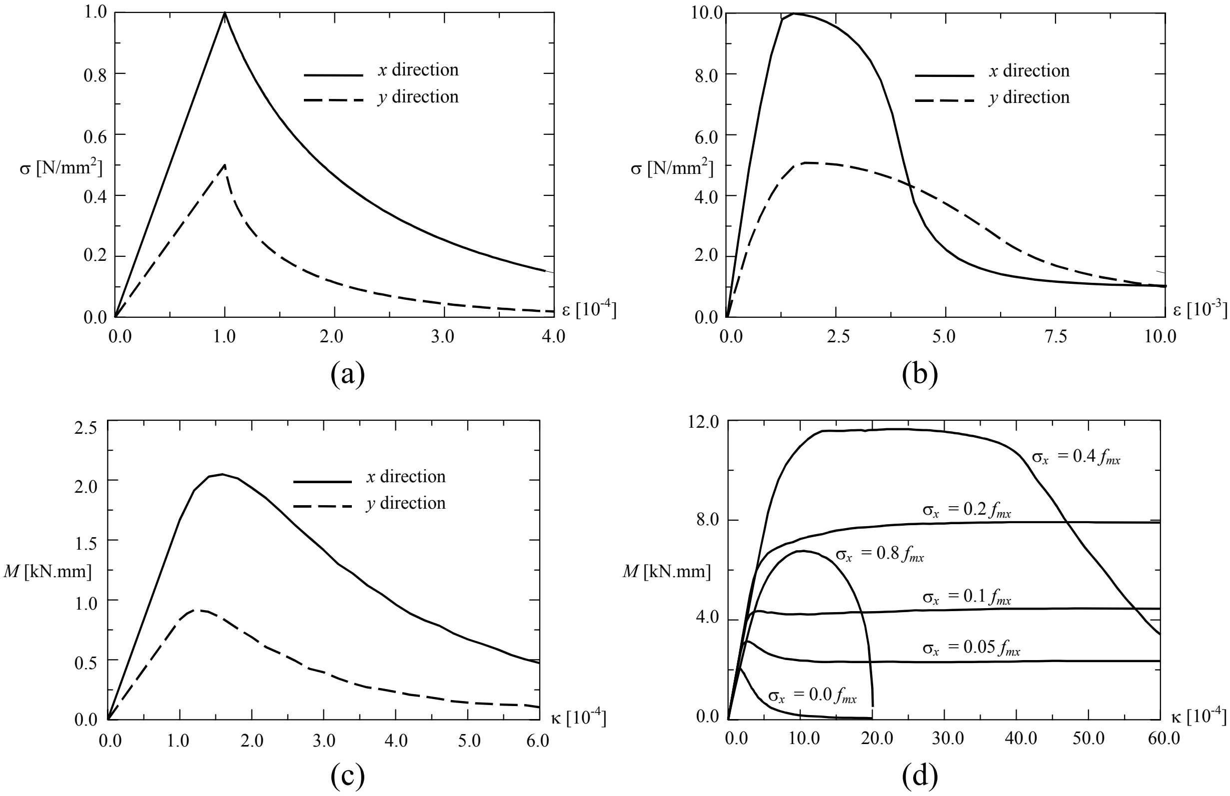
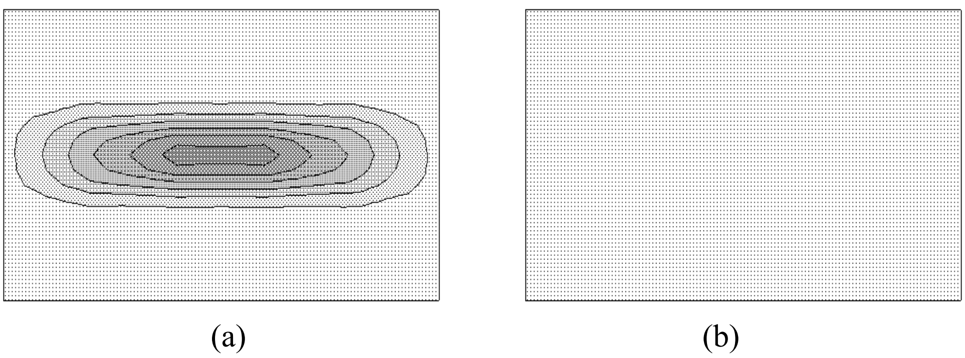
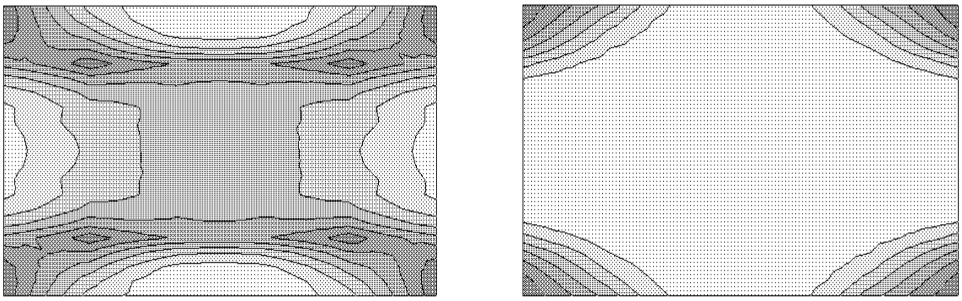


Fig. 2. Possible behaviour of the model along the material axes: in-plane (a) uniaxial tension and (b) compression; out-of-plane (c) pure bending and (d) bending with normal compression



(a) (b)

Fig. 3. Cracking at peak load: (a) bottom and (b) top face



(a) (b)

Fig. 4. Cracking at ultimate load: (a) bottom and (b) top face

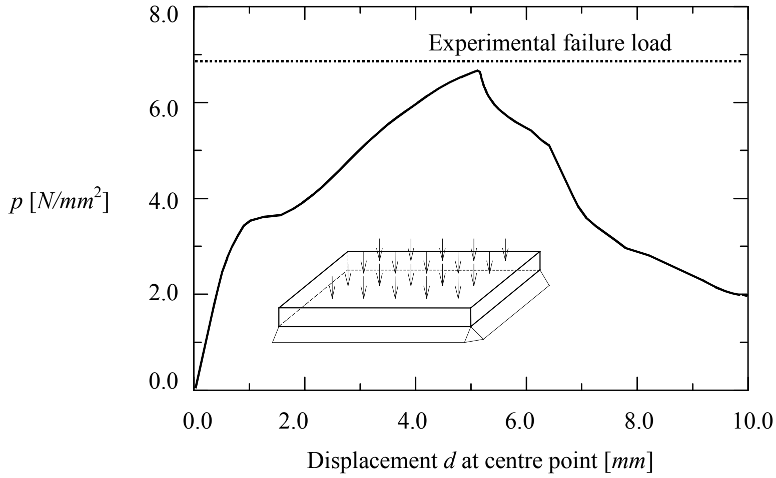
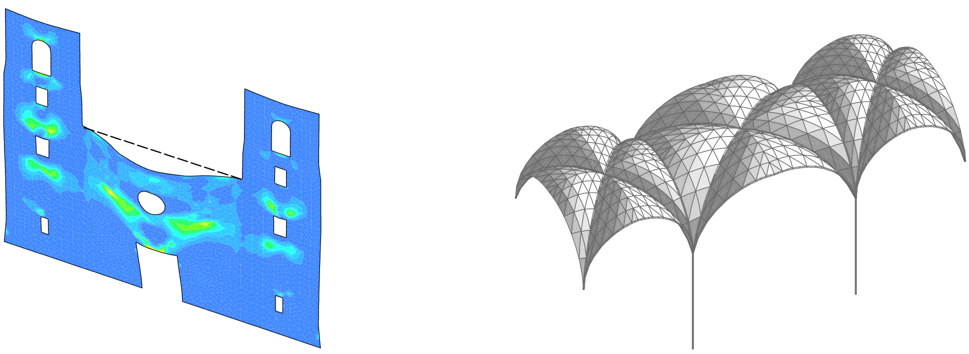


Fig. 5. - Load-displacement diagram and experimental failure load (average of three tests)

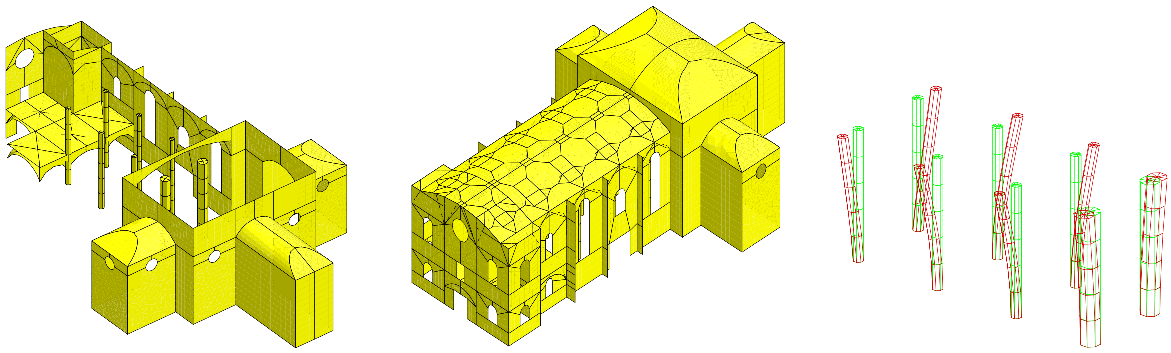
APPLICATIONS

Two applications of the model to engineering problems are illustrated in Figures 6 and 7. The first case aims at explaining existing damage in a façade and choir of a church and the second case involves the safety assessment of a church, with respect to seismic actions.



(a) (b)

Fig. 6. Model of the Holy Christ Church, Outeiro, Portugal: (a) stresses under vertical and horizontal (earthquake) loading and (b) model of the choir



(a) (b) (c)

Fig. 7. Model of the Church of Santa Maria de Belém, Lisbon, Portugal: (a) open southeast view, (b) complete southwest view, (c) deformed mesh for vertical loads (columns)

CONCLUSIONS

A powerful anisotropic continuum model for masonry shells has been presented. The adequacy of the model to reproduce experimental results and its use in engineering applications has been demonstrated.

REFERENCES

de Borst, R. and Feenstra, P.H. (1990), Studies in anisotropic plasticity with reference to the Hill criterion, *Int. J. Numer. Methods Engrg.*, **29**, p. 315-336.

Gazzola, E.A., Drysdale, R.G. and Essawy, A.S. (1985), Bending of concrete masonry walls at different angles to the bed joints, in: *Proc. Third North Amer. Mas. Conf.*, Arlington, Texas, USA, Paper 27.

Li, X., Duxbury, P.G. and Lyons, P. (1994), Considerations for the application and numerical implementation of strain hardening with the Hoffman yield criterion, *Comp. Struct.*, **52**(4), p. 633-644.

Lourenço, P.B. (1997), An anisotropic macro-model for masonry plates and shells: implementation and validation, report 03.21.3.07, Delft Univ. of Tech., The Netherlands.

Lourenço, P.B., Rots, J.G. (1997), A multi-surface interface model for the analysis of masonry structures, *J. Engrg. Mech.*, ASCE, **123(7)**, p. 660-668.

Lourenço, P.B., de Borst, R., Rots, J.G. (1997), A plane stress softening plasticity model for orthotropic materials, *Int. J. Numer. Meth. Engng.*, **40**, p. 4033-4057.

Lourenço, P.B., Rots, J.G. and Blaauwendraad, J. (1998), Continuum model for masonry: Parameter estimation and validation, *J. Struct. Engrg.*, ASCE, **124**(6), p. 642-652.

Owen, D.R.J. and Figueiras, J.A. (1983), Elasto-plastic analysis of anisotropic plates and shells by the semiloof element, *Int. J. Numer. Methods Engrg.*, **19**, 521-539.

Schellekens, J.C.J. and de Borst, R. (1990), The use of the Hoffman yield criterion in finite element analysis of anisotropic composites, *Comp. Struct.*, **37**(6), p. 1087-1096.

Swan, C.C. and Cakmak, A.S. (1994), A hardening orthotropic plasticity model for non-frictional composites, *Int. J. Numer. Methods Engrg.*, **37**, p. 839-860.